The Argonne Pool League Handicap System

(Last revision: 15-Sep-97 by VB and RS)

The Argonne Pool League handicap system is designed to allow “fair” matches between players with different skill levels using an objective and unbiased method. A “fair” match is one in which either player is about equally likely to win based on their average skill levels. Each player has a “skill rating” that is determined from his (or her) win/loss history. The difference in the skill ratings of the two players in a match determines how many games are required by the two players in order to win the match; the higher rated player generally needs more games than the lower rated player, and larger rating differences mean larger differences in the number of games by the two players. Skill ratings range from 0 to over 100. Beginners have ratings in the 20’s or 30’s, more experienced players will be in the 50’s and 60’s, and professional-level players will be over 100.

Initial skill ratings will be assigned by the league. A skill test, which is described in the following section, may be used for new players.

Every time a player plays a match, his skill rating will change. It will increase if he wins, and it will decrease if he loses. The only exception is a player with a rating of zero who loses a match – his rating will remain unchanged. The number of points that a player’s skill rating changes depends on the number of matches he has played.

Number of matches (established players)	Number of matches (new league players)	Skill Rating Change
	1	±6
	2	±4
1	3-5	±3
2-12	6-16	±2
13-up	17-up	±1

At the end of a match, the scorekeeper should indicate on the scoresheet the player’s new skill ratings. In rare situations it may be necessary to adjust a player’s skill rating outside of the normal ratings adjustment. If two or more team captains believe that a skill rating for a particular player is significantly incorrect, then they should notify the other team captains of the situation, and a written notice should be given to the league president; the next time the player in question plays a match, the other team captains, or their representatives, can observe the player. If ⅔ of the team captains agree that there is a rating mismatch, then the player’s skill rating can be adjusted by an amount, not to exceed ±20 rating points, determined by a plurality of the captains.

In the Argonne Pool League, players with higher skill ratings are rewarded by being allowed to play longer matches. An advantage of longer matches is that the game ratio may be chosen to more accurately reflect the two player’s skill ratings. The following charts give the number of games required to win a match by the two players. The skill rating of the stronger player determines which chart is used for the match, or, if both teams agree, shorter charts may be used as a means of speeding up a match. A team total skill rating limit is enforced; this limit is described in the General Rules.

Example: Here is an example of how to determine a matchup and to adjust handicaps. Player A has a skill rating of 55, and player B has a rating of 40. The highest rating, 55, means that Chart-8 should be used. The rating difference of 15 (i.e. 55-40) gives the match length of 4-3. This means that player A must win 4 games to win the match, while player B must win only 3. Suppose that player A, a new player who has played 1 time previously, wins the match, while player B, an established player who has played 14 times this session, loses the match. Player A's new skill rating will be 59 (i.e. 55+4), and player B's new rating will be 39 (i.e. 40-1).

If these same two players were to play again, with their new skill ratings, they would again use Chart-8, but this time the rating difference of 20 (i.e. 59-39) would mean that they would play a 5-3 match, which is slightly tougher for Player A. This shows how the skill ratings for the players adjust after each match until each player has a roughly equal chance of winning the match.

Chart-4 (Highest Skill Rating 0-39)

Rating Difference	Match Games
0-19	2-2
20-up	2-1

Chart-6 (Highest Skill Rating 40-49)

Rating Difference	Match Games
0-10	3-3
11-26	3-2
27-up	4-2

Chart-10 (Highest Skill Rating 70-89)

Rating Difference	Match Games
0-5	5-5
6-14	5-4
15-21	6-4
22-28	5-3
29-36	6-3
37-46	7-3
47-56	6-2
57-62	7-2
63-up	8-2

Chart-8 (Highest Skill Rating 50-69)

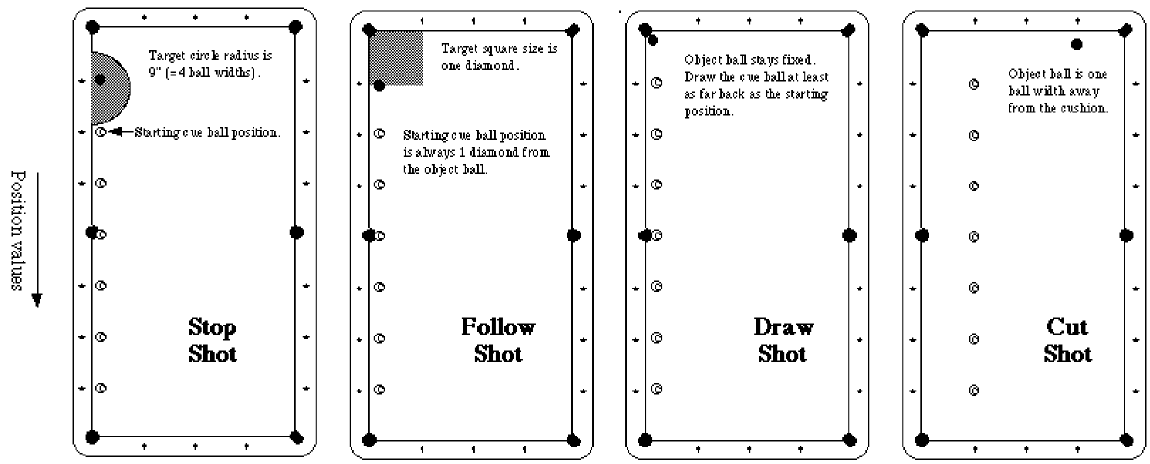
Rating Difference	Match Games
0-6	4-4
7-18	4-3
19-29	5-3
30-39	4-2
40-48	5-2
49-up	6-2

Chart-12 (Highest Skill Rating 90-up)

Rating Difference	Match Games
0-4	6-6
5-11	6-5
12-17	7-5
18-22	6-4
23-28	7-4
29-35	8-4
36-42	7-3
43-48	8-3
49-58	9-3
59-68	8-2
69-74	9-2
75-up	10-2

New Player Skill Assessment: A skill test may be used for the purpose of rating new players. Individual players may also be asked to take a skill test at various times for calibration purposes. When a skill test is taken, the player should endeavor to perform as well as possible during the test.

The test consists of four groups of shots, each of which measures a specific skill. This test should be given to a player after he has warmed up enough to be playing his best and he is accustomed to the equipment characteristics. There are four groups of shots, consisting of a stop-shot group, a follow-shot group, a draw-shot group, and a cut-shot group. The shots are diagrammed for a right-handed player, but the player may switch sides of the table if he chooses. In each case, a success means that the object ball was pocketed correctly and, if appropriate, the cue ball was positioned correctly afterwards. A cue ball scratch counts as a miss.



In the stop-shot group, the goal is to pocket the object ball in the corner pocket and to stop the cue ball within 9 inches of the "ghost ball" position. The object ball is one diamond away from the corner pocket and about an inch or so away from the side cushion. The object ball is always placed in the same position in this shot group, and the cue ball moves farther away from the object ball upon success or closer upon failure. 9 inches is four ball widths, but it may be more convenient to use a hand span for measurements. The cue ball should be close to the side cushion, but not so close as to cause the player difficulty in forming a bridge.

In the follow-shot group of shots the goal is to pocket the object ball in the corner pocket while using topspin on the cue ball to position it within a one-diamond square of the corner pocket without scratching. In this group of shots, both the cue ball and the object ball are moved back farther from the pocket upon success, and moved closer to the corner pocket upon a failure. The initial distance between the cue ball and the object ball is always one diamond. The player is allowed to make small sideways adjustments to the initial cue ball position in order to avoid a scratch.

In the draw-shot group, the goal is to draw the cue ball straight back to at least the starting position. In this group of shots the object ball position remains fixed in the center of the pocket opening and about a ball width from the end cushion, and only the cue ball is moved farther or closer upon success or failure.

In the cut-shot group, the goal is to pocket the object ball in the corner pocket without scratching. The object ball position remains fixed one diamond from the pocket and about one ball width from the end cushion, and the cue ball is moved farther or closer upon success or failure. The cue ball should always be one diamond from the side rail.

The numerical score for each of these tests is determined as follows. The player should begin a group of shots at an estimated starting point that is close to his final score, or at diamond 2 if no estimate can be made. The player should move back one full diamond upon success, and forward one full diamond upon failure, until about three failures have occurred and the player appears to be moving back and forth about his final scoring distance. The player should use a coin on the rail to mark his position before each shot. After the third failure, the cue ball should be moved by half-diamond steps rather than full-diamond steps, and the player should execute four more shot attempts. The average of the starting diamond numbers for these last four shots is then the player's score for that group. The sum of the four scores T for the four groups is then used in the following equation to estimate the skill rating R:

$$R = 2.5\, T + 8$$

For example, suppose that a player scored 4.5, 4.0, 3.5, and 4.0 on the four shot groups; his total score would be T=16.0, which results in an approximate skill rating of R=2.5*16.0+8=48. Round to the nearest integer if necessary. Suppose that another player scored 7.5 on all four shot groups; this corresponds to T=30, and R=83.

This assessment procedure is, of course, only approximate, since it depends only on limited and specific tests of skill. It ignores tactical ability, shot choice decisions, runout ability, and many other aspects of actual game play.

Mathematical Model: The following is a description of the mathematical model used for the above handicap system. It is not necessary to understand this model in order to use the system, but these details are provided for those who are interested. In a match between player 1 and player 2, the probability for player 1 to win an individual game, p_1, is assumed to be related to the skill rating difference according to

$$p_1 = \frac{1}{1 + 2^{-(R_1 - R_2)/30}}$$

or equivalently

$$R_1 - R_2 = \frac{30}{\log 2} \log\left(\frac{p_1}{1 - p_1}\right) = \frac{30}{\log 2} \log\left(\frac{p_1}{p_2}\right)$$

where R_1 and R_2 are the skill ratings of the two players and p_2=1-p_1. If R_1=R_2, then p_1=p_2=0.5 and each player has an equal chance of winning a game. If R_1-R_2=30, then p_1=⅔ and p_2=⅓; this means that player 1 is twice as likely to win an individual game as player 2. If R_1-R_2=60, then p_1=⅘ and p_2=⅕; this means that player 1 is four times as likely to win an individual game as player 2. Every additional 30 points of skill rating difference results in another factor of two in the individual game probability ratio.

If the probability of player 1 winning an individual game is independent of the game score and is given by the value p_1, and if the two players play N=n+m games, then the probability distribution of possible outcomes is given by

$$P(p_1; m, n) = \binom{m+n}{m} p_1^m p_2^n$$

where $P(p_1;m,n)$ is the probability of player 1 winning exactly m games out of the N total, which of course is the same as the probability of player 2 winning n games out of the N total. The binomial coefficient $\binom{m+n}{m} = \frac{(m+n)!}{m!n!}$ is the number of distinct ways to arrive at a given score, and the factor $p_1^m p_2^n$ is the probability, based on the skills of the two players, to arrive at such a score in any particular one of these ways.

For example, suppose that p_1=⅔ and p_2=⅓, and these players play 1 game. The two possible outcomes are P(⅔;1,0)=⅔ and P(⅔;0,1)=⅓, which means that player 1 is twice as likely to win as to lose, as expected. If they play 2 games, then there are three outcomes given by the probabilities P(⅔;2,0)=4⁄9, P(⅔;1,1)=4⁄9, and P(⅔;0,2)=1⁄9.

In a handicapped match in which player 1 is required to win N_1 games and player 2 is required to win N_2 games, the probability of player 1 winning is given by the sum of the probabilities for that player winning in all possible ways. For example, if a match is handicapped at 4-3, then player 1 can win in the three possible ways: 4-0, 4-1, and 4-2. The probabilities of winning in each of these three ways is $p_1P(p_1;3,0)$, $p_1P(p_1;3,1)$, and $p_1P(p_1;3,2)$ respectively, and the probability of player 1 winning the match is the sum of these three terms. In the general case, the probability for player 1 winning a match handicapped at N_1 to N_2 is given by

$$W(p_1;N_1,N_2) = \sum_{k=0}^{N_2-1} p_1 P(p_1;N_1-1,k) = \sum_{k=0}^{N_2-1} \binom{N_1-1+k}{k} p_1^{N_1} p_2^{k}$$

$$= \sum_{j=0}^{N_2-1} p_1^{N_1+j} (-1)^j \left(\sum_{k=j}^{N_2-1} \binom{N_1-1+k}{k} \binom{k}{j} \right)$$

The last equation gives the match win-probability $W(p_1;N_1,N_2)$ as a polynomial function of the player 1 game probability p_1. Solving the polynomial equation $W(p_1;N_1,N_2)-½=0$ numerically for the root in the domain $0 \le p_1 \le 1$ gives the game probability for player 1 that results in a "fair" match of N_1 to N_2. This game probability can then be converted into the following table of skill rating differences which was used to determine the handicap charts. To determine a fair matchup of N_1 to N_2, go to the appropriate row and column of this table; the entry is the rating difference for which both players have exactly a 50% chance of winning that match. For example, a fair 5-4 match would require the long-time average rating difference between the two players to be 10.4. If the actual rating difference between two opponents is higher than that value, then that matchup would favor the higher-rated player; if the actual rating difference is less than that value, then the matchup would favor the lower-rated player. The handicap charts (Chart-4 through Chart-12) were constructed by splitting these differences as fairly as possible between the higher and lower rated players.

Skill Rating Differences for Fair Handicapped Matches

$N_1 \backslash N_2$	1	2	3	4	5	6	7	8	9	10
1	0.0									
2	38.1	0.0								
3	58.3	20.1	0.0							
4	72.1	33.9	13.7	0.0						
5	82.5	44.3	24.1	10.4	0.0					
6	90.9	52.7	32.5	18.8	8.4	0.0				
7	97.9	59.7	39.5	25.8	15.4	7.0	0.0			
8	104.0	65.7	45.6	31.9	21.5	13.1	6.0	0.0		
9	109.3	71.0	50.9	37.2	26.8	18.4	11.3	5.3	0.0	
10	114.0	75.8	55.6	41.9	31.5	23.1	16.1	10.0	4.7	0.0
11	118.3	80.0	59.9	46.2	35.7	27.4	20.3	14.3	9.0	4.3
12	122.2	83.9	63.8	50.0	39.6	31.2	24.2	18.2	12.9	8.1
13	125.7	87.5	67.3	53.6	43.2	34.8	27.8	21.7	16.4	11.7
14	129.0	90.8	70.6	56.9	46.5	38.1	31.1	25.0	19.7	15.0
15	132.1	93.8	73.7	59.9	49.5	41.1	34.1	28.1	22.8	18.0
16	134.9	96.7	76.5	62.8	52.4	44.0	37.0	30.9	25.6	20.9
17	137.6	99.3	79.2	65.5	55.1	46.7	39.6	33.6	28.3	23.6
18	140.1	101.9	81.7	68.0	57.6	49.2	42.2	36.1	30.8	26.1
19	142.5	104.3	84.1	70.4	60.0	51.6	44.5	38.5	33.2	28.5
20	144.8	106.5	86.4	72.6	62.2	53.8	46.8	40.8	35.5	30.7

Suppose that a sample set $\{x_i\}$ of size N is drawn fairly from a large, or infinite, population. The average of the sample set $\bar{x}$ provides an estimate of the true population average $\bar{x}_p$ according to the equation $\bar{x}_p = \bar{x} \pm z_c \frac{\sigma}{\sqrt{N}}$ where σ is the population standard deviation and z_c is the confidence parameter. The confidence parameter is determined by the integral under the standard normal distribution curve; for example, $z_c = 1.96$ for a 95% confidence estimate. This equation says that for a fixed confidence, the estimate of the true population average improves with larger sample sets, and the rate of improvement of that estimate is proportional to $1/\sqrt{N}$. This relation is used as the basis of the ratings adjustment. The match outcomes are treated as a sample space, and the uncertainty in the skill rating is treated as a statistical uncertainty due to the finite sample size. The empirical relation used is $\Delta R_n = \Delta R_1 / \sqrt{N}$, where ΔR_1=6.0 is the estimate of the initial rating uncertainty, and ΔR_n is the rating change after the *n*th match (rounded to the nearest integer). Established league players, those who have played at least one session, are treated as if they have played already 4 matches at the beginning of the season.

Acknowledgments: This handicap system is based on the National Pool League (NPL) handicap system for 9-ball developed by Bob Jewett. Some modifications have been made in order to adapt this method to the Argonne 8-ball league.